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RETRO/IMREL

**A model for transport and land use planning in the
greater Oslo area – version 1.0**

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Summary:

The close relationship between land use and the need for transport infrastructure imply that we can gain by integrating transport- and area planning. This report describes how the Regional transport model for Oslo and Akershus (RETRO, Vold 1999) and Integrated model of residential and employment location in a metropolitan region (IMREL, Boyce and Mattsson 1999) were combined into the transport- and area-planning model for the greater Oslo area RETRO/IMREL (version 1.0). Data obtained for the period around 1998 were used to estimate the model. In a simple model application with two alternative scenarios with respectively tripled fuel tax and quadrupled toll ring charge are compared relative to a reference scenario. The model responses were adequate.

Sammendrag:

Den nære sammenhengen mellom arealbruk og behovet for transport infrastruktur impliserer at vi kan tjene på å integrere transport- og arealplanlegging. Denne rapporten beskriver hvordan den regionale transportmodellen for Oslo og Akershus (RETRO, Vold 1999) og den integrerte modellen for bosted og arbeidsplasslokalisering i et byområde (IMREL, Boyce and Mattsson 1999) ble kombinert i transport og arealbruksmodellen for Stor Oslo – RETRO/IMREL (Version 1.0). Data for perioden omkring 1998 ble brukt for å estimere modellen. I en enkel anvendelse av modellen ble to alternative scenarier med henholdsvis tredoblet bensinavgift og firedoblet bomavgift sammenlignet relativt til et referansescenario. Resultatene fra anvendelsen viste adekvat modellrespons.

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Preface

This working report was financed by the strategic program *Social-economic Analysis of Infrastructure Investments* at the Institute of Transport Economics and by the project *Development of RETRO/IMREL for greater Oslo*. Dr scient Arild Vold accomplished the research work described in this report. Fruitful discussions with Harald Minken, Lars Göran Mattson, Daniel Jonsson and Tom E. Markussen contributed to the work. Odd I. Larsen was responsible for quality assurance. Laila Aastorp Andersen did the secretary work.

Oslo, December 2000
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Summary:

RETRO/IMREL

A model for transport and land use planning in the greater Oslo area – version 1.0

I. Background

Urban development changes the demand of land for residential or industrial purposes and the requirements for transport supply. New transport infrastructure or modification of the existing transport infrastructure is part of overall planning of land use changes both in urban and rural areas. The close relationship between land use and transport demand implies that we can benefit by the integration of transport- and land use planning.

City planners and politicians may use a number of instruments in planning strategies to affect the developments, where the strategies can be more or less sustainable ways of maximisation of economic efficiency. Instruments that can be used as part of strategies to achieve goals for efficiency and sustainability includes regulation of areas for new purposes, establishment of new transport infrastructure or modification of the existing transport infrastructure. Available instruments in official land use and transport planning also includes movement of workplaces in government and municipalities and pricing instruments for planning purposes (e.g. road pricing or property taxes).

II. Objective

This report describes how the Regional transport model for Oslo and Akershus (RETRO, Vold 1999) and the Integrated model of residential and employment location in a metropolitan region (IMREL, Boyce and Mattsson 1999) were combined into the Transport- and land use planning model for the greater Oslo area – RETRO/IMREL (version 1.0). The report includes short descriptions of RETRO and IMREL and describes how IMREL was modified and combined with RETRO.

III. RETRO and IMREL

RETRO is a four-stage transport model programmed in the C programming language that can be used for periods with peak and off peak traffic load. It includes route choice, destination choice, mode choice and travel frequency (Vold 1999). The route choice is calculated with EMME/2 for a real network covering

438 zones in Oslo and Akershus. A nested logit model (i.e., frequency, destination and model choice) calculates travel demand. RETRO is connected to a sub model for car ownership.

The original IMREL model consists of a submodel for residential location (RES) and a submodel for employment location (EMP). RES can be expressed in terms of a bilevel optimisation problem. There is one worker per household in RES, and each household requires one housing unit. EMP is of the multinomial logit type with a linear systematic component with variables for the location conditions in the employment zone, the rent level and accessibility to the work force.

IV. RETRO/IMREL

The RETRO/ IMREL model is designed to assess the effects of land use and transport policy instruments, where the effects are calculated relative to a reference scenario. Available policy instruments in RES are: (1) the relative changes in the number of area units that are regulated for housing supply in the zones and (2) upper and lower limits for the share of residences in the zones as relative to the shares in the respective zones in the reference scenario. Policy instruments in EMP are (1) the relative change in the total area available for employment location, (2) the number of pupils (17-19 year) in school, and (3) the number of students in universities. Vold (1999) describes the available policy instruments in RETRO.

Although the first version of RETRO/IMREL does only take into account the periods of peak traffic load, the RETRO sub model can be used to calculate the resulting off peak transport costs and demand. Variables for disutility, housing price in the zones and the transport costs between zones are all endogenously calculated in RETRO/IMREL. Disutility, housing price and transport cost are important factors in RES. There are many variables that affect disutility in the positive or negative direction. In the present version of RETRO/IMREL, disutility in a zone is only affected by population density in areas zoned for housing supply. We express density as the number of households per area unit that are regulated for housing supply.

In model applications, when the three sub models are run in loop, the sub models interchange data. The RETRO sub model use data for residential location and employment location from RES and EMP, respectively. The employment location sub model use data of residential location from RES and some exogenous data from RETRO, whereas RES uses data of employment location and some exogenous data from RETRO.

The model implementation of RES and EMP are based on the C-version of the constrained optimisation algorithm DONLP2 (7/99), which is developed by P. Spellucci, Technical University at Darmstadt. The 1st level problem is represented according to the input specifications of DONLP2.

V. Data and model estimation

Input data and data for estimation of the RETRO/IMREL model were collected for the period around 1998. The data describes population subgroup sizes, the total number of jobs, the number of jobs that attracts people (grocer stores, service and restaurants) and data for the total shopping centre area. Data that are necessary as input to RES includes: (1) area available for residential purposes, (2) transportation costs and (3) travel demand and (4) upper and lower limits on the proportions of the total population that can live in the different zones. The input data to EMP includes (1) attraction indicators for employment location in the zones and (2) area available for work place location and (3) the maximum number of workplaces per area unit available for workplace location.

The attraction indicators constitute a measure for the accessibility to the work force and several data, corresponding to policy instruments, that describe the attractions in the zone. The measure for accessibility to the work force includes data for the transport costs (NOK) between all pair of zones in the area and the share of housing supplies in the zones.

Model estimation is based on methods that adjust parameters such that data and corresponding endogenous model variables fit. For RES, the endogenous model variables are the share of households in the zones, and for EMP the endogenous variables are the share of workplaces in the zones. For estimation we needed additional data for household- and employment location for 1998 for the 49 zones (27 municipalities in Oslo and 22 counties in Akershus) and an OD matrix for the total number of trips in the peak period in the base scenario.

The methods for parameter estimation did not require iterated interactions between the sub models while estimating the parameters. For estimation of RES, the method of maximum likelihood a dual formulation of RES was applied. The Nelder-Mead simplex algorithm for optimisation of non-linear functions was used for optimisation. Linear regression was used to estimate parameters in EMP.

In order to apply the model for a future time period, input data for the future time period is needed.

VI. Model application

A simple model application is presented, where two alternative scenarios are simulated and compared relative to a base scenario, where the base scenario describes the situation in the period around 1998. One of the alternative scenarios differs from the base scenario in that the fuel tax is tripled, and the other alternative scenario differs in that the toll ring charge is quadrupled. Moreover, the total number of residences and workplaces in the base scenario and the alternative scenarios are equivalent.

In the scenario with a tripled fuel tax, both the number of residences and the number of workplaces increase in areas close to the city centre. However, there is an insignificant increase in the number of residences in the city centre, where the

area for residential purposes is small and the number of residences is small in the base scenario (ca. 1000 residences). The number of workplaces in the city centre increases by 30% relative to the base scenario. Also in the scenario where toll charges are quadrupled, we obtain a centralisation in areas that are close to the city centre. Different from the scenario with an increase in the fuel price is that we also get migration of households and work places to the Asker municipality some distance south west of Oslo. This is because more households and work places inside the toll ring increases the disutility in these zones, and a share chooses instead to establish in Asker instead in which there are many residences and workplaces initially in the base scenario.

VII. Conclusion

As prices change, it takes time before household- and workplace location again is in equilibrium. RETRO/IMREL calculates the long-term effects of changed prices. This means that the results are for a future year, where household- and employment locations have reached a new equilibrium. We can not say how long time it will take before the new equilibrium is reached. The time horizon depends on how narrow we set the intervals that are bounded by the upper and lower bounds on shares of residential - and employment location and the size of the areas available for different purposes. In real life it seems reasonable that at least 10 years is needed to capture a significant trend.

This report describes RETRO/IMREL - a combined land-use and transport model for the greater Oslo area and applications on simple test cases. Equilibrium models for transport- and land use planning like RETRO/IMREL are not expected to be capable to assess a very precise forecast for the amount of relocation among the zones relative to a base case. It is expected, however, that results from models of this type are reasonable and that the trends are reliable.

The qualitative and quantitative model behaviour in the test cases we have presented in this report seems reasonable. The cases were simple, however, so further development, validation and use is required in order to obtain more interesting results.

For future work, it would be interesting to include exogenously given variables in the disutility function. Interesting in this respect would be to include a variable that represents the service level, the availability for parking places, external environmental costs and a property tax. New variables for description of the establishment of workplaces would also be interesting. For this purpose it would be interesting to include an endogenously calculated variable that express the concentration of workplaces or the closeness to other workplaces.

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1 Introduction

Most cities are subject to economic growth, demographic changes and technological changes in the industries. These developments in cities change the demand for land for residential or industrial purposes and the requirements of transport supply.

Land use planning includes zoning of activities, and altered land use changes the need for transport infrastructure. The close relationship between land use and the demand for transport infrastructure imply that we can benefit from integration of transport- and land use planning. City planners and politicians may use a number of instruments in planning strategies to affect the developments, where the strategies can be more or less sustainable ways of maximising economic efficiency. Instruments that can be used to achieve goals for efficiency and sustainability includes (1) zoning of areas for new purposes, (2) establishment of new transport infrastructure or (3) modification of the existing transport infrastructure. Available instruments in official land use and transport planning also includes (4) movement of workplaces in government and municipalities and (5) pricing instruments for planning purposes (e.g. road pricing or property taxes).

It is difficult, however, to understand how the available instruments affect land use, transport demand, emissions from transport, economic efficiency in the transport sector and the overall welfare. In the AFFORD project, the RETRO¹ model was used to assess the effects of first-best and second-best road pricing strategies on travel behaviour and car ownership, (Vold, Minken and Fridstrøm, 1999 and Fridstrøm et al. 2000). The RETRO model does not include effects on land use changes, however. Thus the effects were only covering the short- to medium term time horizon.

The IMREL² model can be used to calculate the effects that general land use - and transport policy instruments - have on land use, transport supply and travel demand. The original IMREL includes sub models for residential location (RES), employment location (EMP) and a network model for route choice and destination choice for work trips. However, it does not include variable mode choice and trip frequency. RETRO includes route choice, mode choice destination choice and trip frequency in periods of peak and off peak traffic load, and is supplemented with a car ownership model. Ramjerdi, Jensen and Rand (1995) proposed to integrate RETRO and IMREL. By connecting IMREL and RETRO it would be possible to extend the range of policy instruments for which the short-, medium- and long run effects can be calculated.

This report describes how the RETRO and IMREL models were combined into the transport- and area-planning model for the greater Oslo area – RETRO/IMREL (version 1.0). The report describes data and methods that were used for estimation and calibration of parameters and results from a simple model application. Chapter 2 describes IMREL,

¹ Vold (1999) describes the RETRO model.

² Anderstig and Mattsson (1991) and Boyce and Mattsson (1999) describe the IMREL model.

Chapter 3 describes RETRO and Chapter 4 describes how IMREL is modified and combined with RETRO. Data that are applied as model input and for estimation of the present version of RETRO/IMREL are described in Chapter 5 and the solution algorithm and implementation issues are described in Chapter 6. Parameter estimation of the RETRO/IMREL model is described in Chapter 7, where estimates of parameters in RES and EMP and the statistics on how well the model and data fits are presented and discussed. The model application is presented in Chapter 8. Two alternative scenarios are simulated and compared relative to a base scenario. The base scenario refers to the situation around 1998. One of the alternative scenarios differs from the base scenario in that the fuel tax is tripled, and the other alternative scenario differs in that the toll ring charge is quadrupled. Chapter 9 gives final conclusions and some notes and recommendations for future work. Appendix A contains the general optimisation algorithm that was used for maximum likelihood estimation of RES.

2 The original IMREL

Anderstig and Mattsson (1991) developed the original IMREL model, which consists of a submodel for residential location (RES) and a submodel for employment location (EMP). RES assesses the share of housing supply h_i in the zones $i=1, \dots, n$ and EMP assesses the share $w_j, j=1, \dots, n$ of working places in the zones. The lower l_i and upper u_i bounds on the share of housing supply h_i are exogenously given, whereas the market clearing housing price $q_i(\mathbf{h})$, the housing disutility $d_i(\mathbf{h})$ and the cost perceived by the traveller on trips between zone i and j $c_{ij}(\mathbf{h})$ depends on the housing supply $\mathbf{h} = [h_1, \dots, h_n]$.

RES is based on the assumption that there is one worker per household, and that each household requires one housing unit. The following function is used as a welfare criterion (Boyce and Mattsson, 1999) for the location of housing:

$$V(\mathbf{h}) = \frac{1}{\mu} \sum_j w_j \ln \sum_i \exp[-\mu(q_i(\mathbf{h}) + d_i(\mathbf{h}) + c_{ij}(\mathbf{h}))] + \sum_i h_i q_i(\mathbf{h})$$

The first term represents consumer surplus of the employees, resulting from housing location pattern \mathbf{h} , and the second term represents the producer surplus associated with pattern \mathbf{h} .

RES can be expressed in terms of a bilevel optimisation problem that consist of an upper (U ;) and a lower (L ;) optimisation problem. The upper problem is:

$$\begin{aligned} U : \quad & \max_{(\mathbf{h})} V(\mathbf{h}), \\ \text{s.t.} \quad & l_i \leq h_i \leq u_i \quad \forall i, \\ & \sum_i h_i = 1 \end{aligned}$$

where $\mathbf{h} = [h_1, \dots, h_n]$. The market clearing housing prices $q_i(\mathbf{h})$, the generalised travel costs between residential zone i and workplace zone j , as perceived by the traveller $c_{ij}(\mathbf{h})$ and the housing disutility $d_i(\mathbf{h})$ $i, j = 1, \dots, n$ solve the lower optimisation problem which is a real network distribution and user equilibrium problem (similar to problem 7.15a in Sheffi, 1985).

The objective function of the lower optimisation problem Z_1 includes a term that ensures user equilibrium and an entropy term that govern the distribution of trips:

$$\begin{aligned}
 L: \quad & \min_{T_{ijr}, P_{ij}} Z_1 = \frac{1}{T} \sum_a \int_0^{f_a} c_a(\mathbf{x}) dx + \sum_i \sum_j P_{ij} \cdot d_i(\mathbf{h}) + \frac{1}{\mu} \sum_i \sum_j P_{ij} \ln P_{ij}, \\
 s.t \quad & \sum_r T_{ijr} = TP_{ij} \quad \forall ij, \\
 & \sum_j P_{ij} = h_i \quad \forall i, \\
 & \sum_i P_{ij} = w_j \quad \forall j, \\
 & T_{ijr} \geq 0 \quad \forall ijr.
 \end{aligned}$$

where μ is a parameter, c_a is the travel costs on network link a , T_{ijr} is the number of work trips between zone i and j on route r . The market clearing housing prices $q_i(\mathbf{h})$, $i = 1, \dots, n$, becomes a vector of Lagrange multipliers in the lower optimisation problem that are unique up to an additive constant (see Boyce and Mattsson, 1999). We also have that the flow on link a is $f_a = \sum_{ijr} T_{ijr} \cdot \delta_{ijr}^a$, $\forall a$, where $\delta_{ijr}^a = 1$ if link a belongs to route r and 0

otherwise. The housing disutility $d_i(\mathbf{h})$ can be a function of the negative characteristics of density but also characteristics of the zone that are favourable. In this respect, it is well known that the negative sides of the densely populated areas in the inner city of Oslo is outweighed by the fact that the areas are attractive because of the high level of service and high accessibility to workplaces and service in circumscribed areas. Accordingly, areas that have high disutility due to high density may still be attractive due to location advantage, which is reflected in the market clearing housing prices.

The necessary conditions of a solution of the lower optimisation problem are:

$$\begin{aligned}
 (2.1) \quad & T_{ijr} \left(\frac{1}{T} \sum_a c_a(f_a) \cdot \delta_{ijr}^a - u_{ij} \right) = 0 \quad \forall ijr \\
 & \ln P_{ij} = \mu \cdot (-q_i + r_j - d_i(\mathbf{h}) - c_{ij}) - 1
 \end{aligned}$$

where c_{ij} is the travel costs per work day for work trips between zone i and zone j . The housing price per day q_i , the income per day r_j , and u_{ij} are all Lagrange multipliers. Boyce and Mattsson (1999) shows that the proportion of workers living in zone i and working in zone j , P_{ij} , becomes

$$(2.2) \quad P_{ij} = w_j P_{i|j},$$

where

$$(2.3) \quad P_{i|j} = \frac{\exp[-\mu(q_i(\mathbf{h}) + d_i(\mathbf{h}) + c_{ij}(\mathbf{h}))]}{\sum_i \exp[-\mu(q_i(\mathbf{h}) + d_i(\mathbf{h}) + c_{ij}(\mathbf{h}))]}.$$

Anderstig and Mattsson (1991) originally described EMP. In general terms the probability that a workplace is located in employment zone j is quantified by the multinomial logit formula:

$$(2.4) \quad w_j = \frac{e^{\alpha A_j + \sum_k \beta_k \cdot I_{kj} - \rho_j}}{\sum_l e^{\alpha A_l + \sum_k \beta_k \cdot I_{kl} - \rho_l}}$$

where I_{kj} are indicators related to the location conditions in employment zone j and are independent on the location of the households, ρ_j is a (positive or negative) rent (shadow price) associated with the location of a workplace to employment zone j , and α and β_k are parameters to be estimated. The accessibility to the labour force depends on the location pattern of the households and travel costs. The accessibility measure in EMP is defined as

$$A_j = \log \sum_i h_i \left(\sum_m e^{-a_m - b_m c_{ijm}} \right)^{\lambda-1},$$

where a_m and b_m are parameters.

3 RETRO

RETRO is a four-stage transport model that can be used for peak and off peak traffic load. It includes route choice, destination choice, mode choice and travel frequency (Vold 1999). RETRO is connected to a sub model for car ownership that is also implemented in the C programming language. The sub model for route assignment and travel costs is implemented in EMME/2. The route choice is calculated with EMME/2 for a real network covering 438 zones in Oslo and Akershus, and travel demand is calculated between 49 zones by a nested logit model (i.e., frequency, and destination and model choice). Transformation matrix transforms travel demands between the 49 zones to the 438 zones and vice verses for travel costs. The mathematical programming formulation of RETRO (Vold, 1999) is:

$$\min_{f_a, f_{rs}} z(f_a, f_{rs}) = \sum_a \int_0^{f_a} c_a(\omega) d\omega - \sum_{rs} \int_0^{f_{rs}} D_{rs}^{-1} d\omega$$

subject to

$$\sum_k f_k^{rs} = f_{rs}, \forall r, s,$$

$$f_k^{rs} \geq 0 \forall k, r, s,$$

$$f_{rs} \geq 0 \forall r, s,$$

$$f_{rs} \leq \bar{q} \forall r, s,$$

$$f_a = \sum_{rs} \sum_k f_k^{rs} \delta_{a,k}^{rs} \forall a,$$

where the elements, f_a , $a=1, \dots$, represents traffic flow on the road network links, c_a is the generalised cost of travel on network link a , f_k^{rs} is traffic flow on route $k \in K$ between OD-pairs rs , f_{rs} is the demand for car trips between OD pairs rs ($T P_{ij}$), and the upper bound \bar{q} is added for computational reasons. We have that $\delta_{a,k}^{rs} = 1$ if link a is part of a route k between the OD pair rs and zero elsewhere. The demand for trips by car is expressed in terms of the nested logit model $f_{rs} = \sum_{i=1}^N W_i \cdot E_i[q_i] \cdot P_i(m | sq_i) \cdot P_i(s | q_i)$.

Here N is the number of sampled individuals in origin zone r , $E_i[q_i]$ is the expected trip frequency of individual n and W_i are corresponding weights on individual n . Further, $P_i(m | sq_i) \cdot P_i(s | q_i)$ denotes the probability that a trip by individual n residing in zone r makes a trip to destination s by mode m . Parameters values are found in Vold (1999), where a detailed description of the implementation for the RETRO model is given. In addition, inelastic trips from external zones (zones outside of Oslo and Akershus) are added.

4 Connecting RETRO and IMREL

In order to obtain a consistent combination of RETRO and IMREL, some modifications must be made in IMREL. Section 4.1 and 4.2 below describes the modifications that were necessary to obtain a fully integrated and consistent combination of RETRO and IMREL. The first version of RETRO/IMREL does only take into account the periods of peak traffic load. However, RETRO can be used to calculate the effects that land use changes have on off peak transport costs and demand.

There is a close relationship between the exogenous variables that are present in a model and the land use and transport policy instruments that can be used in the model. The level on instruments can be expressed in terms of policy variables. Hence, for convenience policy variables are used as part of the model input to RETRO/IMREL. In addition, the exogenously given variables in RES and EMP, that is, travel costs and travel demand are obtained from RETRO, where the travel costs are affected by policy variables that are part of the model input to RETRO (Vold, 1999). Hence, policy variables in RETRO implicitly affect residential – and household location and vice versa.

The RETRO/ IMREL model is capable of calculating the effects of land use and transport policy instruments relative to a base scenario.

4.1 Modifications of RES for connection with RETRO

Both RETRO and IMREL assess travel demand P_{ij} and travel costs c_{ij} between zones i and j . In order to deal with this we use RETRO to calculate P_{ij} and c_{ij} , and modify IMREL by reformulation of P_{ij} and c_{ij} as exogenously given variables. To do this, we remove the lower optimisation problem in RES. Instead the RETRO model is applied for calculation of c_{ij} and P_{ij} .

It is noticed that the original IMREL model does only assess work trips, whereas the shares P_{ij} assessed by RETRO include all trips. RETRO calculates the total demand for car trips in the peak and off peak periods. Not all of these trips are work trips. In the peak period it is assumed that the shares of work trips, business trips and other trips by car are 33.44 %, 24% and 42.56%, respectively. Furthermore, it is assumed that 44% percent of the peak trips by public transport are work trips, 56% are other trips and 0% are business trips. Thus it makes no sense to include the constraint $\sum_i P_{ij} = w_j \quad \forall j$ in RETRO. This means that r_j in the second equation of the constraints (2.1) can be set at zero. The first equation becomes unchanged.

We let c_{ij} be the generalised costs of travel between two zones expressed as the logsum of travel cost for travelling with the three modes. The RETRO model calculates this as (Vold, 1999).

$$(4.1) \quad c_{ij} = \frac{1}{\mu_m} \ln \sum_{m \in \mathbf{M}_{rd}} e^{(\tilde{V}_m^{ij} + \tilde{V}_{md}^{ij}) \cdot \mu_m}$$

where \tilde{V}_m and \tilde{V}_{md} are the systematic components for discrete travel choice in peak periods in the RETRO model, and only the ratio of the relationship $\frac{\mu}{\mu_m}$ can be identified from data.

Hence an estimate of μ is intimately dependent on the value of μ_m .

In RETRO/IMREL the market cleared housing price is not a Lagrange multiplier. Instead it is defined by:

$$(4.2) \quad q_i = -d_i(\mathbf{h}) - \sum_{j=1}^n (P_{ij} \cdot c_{ij}) + \phi_i, \quad \forall i$$

We see that q_i depends on the disutility $d_i(\mathbf{h})$ of living in zone i , a measure of transportation costs, and a zone specific constant ϕ_i that reflects costs and benefits not included elsewhere.

We use this expression for q_i in the upper optimisation problem of IMREL and replace the lower optimisation problem of RES by the mathematical programming formulation of RETRO. The 1st level of RES now becomes:

$$(4.3) \quad \begin{aligned} 1^{st} \text{ level: } \max_{(h)} V(h) &= \frac{1}{\mu} \sum_j w_j \ln \sum_i \exp(-\mu_i \cdot (\sum_k (P_{ij} \cdot c_{ik}) + c_{ij} + \phi_i)) + \\ &\quad \sum_i h_i \cdot (-d_i(h_i) - \sum_j (P_{ij} \cdot c_{ij}) + \phi_i), \\ s.t. \quad l_i &\leq h_i \leq u_i, \quad \forall i, \\ \sum_i h_i &= 1 \end{aligned}$$

Now RES responds to transport costs c_{ij} , travel demand P_{ij} , the number of workplaces, w_j , and disutility d_i . Transport costs and travel demand are exogenously given from the RETRO model, whereas the number of workplaces is given from EMP. Transport policy instruments that can be applied in RETRO affects both transport costs and travel demand, and the number of workplaces is affected by instruments applied in EMP. Hence, residential location is indirectly affected by the measures applied in RETRO and EMP.

4.2 Disutility and bounds on housing

There are many variables that affect disutility in the positive or negative direction. The disutility in a zone is obviously affected by population density in areas regulated for housing supply. We denote the total number of people in the region of Oslo and Akershus by TP and the area units that are regulated for housing supply in zone i by RH_i . We assume

that h_i represents the share of the population that is located in zone i , and that the relationship between the number of people, TP , and the number of households, TH , is the same in all zones. We express density as the number of households per area unit that are regulated for housing supply. Hence the housing density in areas regulated for housing supply in zone i is expressed by

$$DP_i(h_i, TH, RH_i) = (h_i TH)/RH_i.$$

Both TP and TH are exogenous variables in RETRO/IMREL that change over time. In Oslo of January 1, 1998, the number of households was 291254 (Oslostatistikken, 1999, Table 2.12) and the number of households in Akershus of 1998 was 198000 (Statistics Norway, 1998).

The service level also affects the disutility in a zone. We may express the service level in terms of the number of people that have jobs that attracts people S_i , in a zone. High service level can be either positive or negative. It is positive in the sense that it makes the life easier for the inhabitants, but negative in the sense that this gives high transport activity and hence high environmental costs. Besides this, we may include a component for the transport activity, expressed as the total distance by car CD_i as well. The total driving distance by cars in the zones can be used as a proxy for the perceived environmental costs in the zones. The number of parking places per inhabitant in the zone, RP_i , can also affect the disutility.

In order to investigate the effects of certain transport- and land use measures on the disutility, there must be variables in the disutility function through which the measures can be expressed. We can use measures in terms of policy variables that change the relative sizes on RH_i , S_i , CD_i and RP_i (Table 5.1).

In RETRO (Vold, 1999) the relative changes in model variables are expressed in terms of policy variables. With the policy variables x_{1i} and x_{2i} , the disutility function becomes:

$$(4.4) \quad d_i = \zeta_1 \cdot DP_i(h_i, TH, x_{1i} \cdot RH_i) - \zeta_4 \cdot CD_i + x_{2i} \cdot \zeta_3 \cdot RP_i + \zeta_2 \cdot S_i$$

where $\zeta_1, \zeta_2, \zeta_3$ and ζ_4 are parameters that must be estimated.

Anderstig and Mattsson (1991, p.181) set the lower bounds on the zonal supplies of housing at 70 percent of the number of households in the zone according to the regional plan proposal, and no upper bound was applied. We will deviate somewhat from this, however. Instead we rely on a relationship between the upper bound, u_i , for the share of housing in zone i and the area regulated for housing supply RH_i . The relationship depends on how close the houses can be built, and the number of floors in blocks of flats. We suggest the following formula for description of this relationship:

$$u_i = \frac{\tilde{h}_i \cdot (1 + x_{4i})}{RH_i} \cdot (RH_i + RH_i \cdot x_{1i})$$

where x_{1i} and x_{4i} , $i = 1, \dots, n$ are policy variables for the zones and \tilde{h}_i is the housing share in zone i in the base year. Here x_{1i} is the relative change in the number of area units that are regulated for housing supply RH_i in zone i , and x_{4i} is the upper limits for the share of

residential location in zone i per area unit as relative to the shares in zone i in the base scenario $\frac{\tilde{h}_i}{RH_i} \cdot (1 + x_{i4})$.

Parameters in the disutility function can be interpreted as the "willingness to pay" for each unit of increase or reduction in the corresponding variables. These parameters can be determined from the literature. In this version of RETRO/IMREL, however, we do not change the service level, or the number of parking places per inhabitants in the zone. Thus, these components are assumed represented by the zone specific parameter. As it is somewhat difficult to use EMME/2 to subdivide the total driving distance by car on the zones, we neither include this variable in the model. Thus, only x_{1i} and x_{4i} are available policy variables in the present version of RETRO/IMREL (Table 4.1).

When x_{1i} is changed relative to the base scenario, it is important to be aware that the area for residential purposes plus the area for workplace location and the area for other purposes must not exceed the total area of the zone.

Table 4.1. Policy variables in the present version of RES

x_{1i}	Relative change in the number of area units that are regulated for housing supply in zone i , $x_{1i} \cdot RH_i$.
x_{4i}	Upper limits for the share of residential location in zone i per area unit as relative to the shares in zone i in the base scenario $\frac{\tilde{h}_i}{RH_i} \cdot (1 + x_{i4})$.

4.3 Modifications in EMP for connection with RETRO

Equation (2.4) can be used to determine $w_s = w_s(\mathbf{h}, c_{1s}, \dots, c_{ns}) \quad \forall s$ endogenously as a function of \mathbf{h} and c_{1s}, \dots, c_{ns} and other exogenously given variables. The equation includes the shadow price ρ_j of the rent. In order to avoid the problem of obtaining estimates for this shadow price, we can reformulate the employment model as the solution of a mathematical programming problem, of which the solution includes values of the shadow price that are endogenously determined.

The mathematical programming problem is

$$\begin{aligned}
 \min_w f &= \frac{1}{\gamma} \sum_j (w_j \ln(w_j) - w_j) - \sum_j M_j \cdot w_j \\
 (4.5) \quad w_j - w'_j &\geq 0 \quad \forall j \\
 w''_j - w_j &\geq 0 \quad \forall j \\
 \sum_j w_j &= 1
 \end{aligned}$$

where w'_j and w''_j can be set at lower and upper bounds. The Lagrangian of this problem is

$$L = f - \sum_j \rho_{2j} \cdot (w_j'' - w_j) + \rho_{1j} \cdot (w_j - w_j')$$

Hence, the first order conditions are given by

$$\frac{dL}{dw_j} = \frac{1}{\gamma} \ln(w_j) - \frac{1}{\gamma} - \rho_{2j} + \rho_{1j} = 0 \quad \forall j$$

$$\rho_{1j} \cdot (w_j - w_j') = 0 \quad \forall j$$

$$\rho_{2j} \cdot (w_j'' - w_j) = 0 \quad \forall j$$

$$\rho_{1j}, \rho_{2j} \geq 0 \quad \forall j$$

The first order conditions can be used in a non-linear programming algorithm in order to obtain a solution of the mathematical programming problem with respect to values of ρ_{1j} , ρ_{2j} and w_j .

If we let

$$M_j = \alpha \cdot A_j + \sum_k \beta_k \cdot I_{kj},$$

it is easy to shown that the first order conditions imply that the solution is equivalent with equation (2.4) which is a multinomial logit type. We may multiply f by γ . By doing this, we do not need to estimate this parameter explicitly, because it becomes only a factor in M_j .

As we want to make the accessibility to the work force based on data from RETRO and RES, we slightly modify the accessibility measure A_j in a way that makes it consistent with the logsum for generalised travel costs of equation (4.1):

$$(4.6) \quad A_j = \ln \sum_i h_i \cdot \left(\sum_{m \in M_{nd}} e^{(\tilde{V}_m^{ij} + \tilde{V}_{md}^{ij}) \cdot \mu^m} \right)$$

Hence, exogenously given variables that affects employment location includes household

location from RES and travel costs $c_{ij} = \frac{1}{\mu_m} \cdot \ln \left(\sum_{m \in M_{nd}} e^{(\tilde{V}_m^{ij} + \tilde{V}_{md}^{ij}) \cdot \mu^m} \right)$ from RETRO.

Additionally some exogenously given data can be included as indicators I_{kj} . Available data for this purpose includes the number of metro stations in the zones I_{1j} , the number of railway stations I_{2j} , total area (or total area (km²) available for employment location) I_{3j} , the share of employees that have free parking I_{4i} , the number of pupils in school (13-16 year) I_{5i} , and (17-19) I_{6i} , the number of students at Universities I_{7i} and total area with transport infrastructure I_{8i} . Only A_j and the indicators I_{3j} , I_{6j} I_{7j} became significant in the linear regression analysis for estimation of the parameters (see section 7.2).

In a future version of IMREL it would be interesting to also incorporate an accessibility measure for the closeness to other workplaces, which could be endogenously determined.

Input data that are needed to run EMP includes the upper, w_j'' , and lower, w_j' , bounds on the number of workplaces in the zones. These limits can probably be calculated on the basis of data for the area available for employment location, I_{3i} , and the number of

workplaces per area unit available for employment location, WA_i . A suitable equation for calculation of the upper number of workplace locations is:

$$w_i^u = y_{3i} \cdot \frac{I_{3i}}{\sum_i y_{3i} \cdot I_{3i}} \cdot y_{2i} \cdot \frac{WA_i}{WA_{max}},$$

where WA_{max} is a parameter, y_{2i} and y_{3i} are zone specific policy variables, WA_{max} is the maximum of WA_i , and we suggest that I_{3i} and WA_i are calculated on the basis of actual values in the base year.

The variables in EMP that can be potentially connected to policy variables include the variables that are part of the workplace location attraction measure, M_j . The accessibility measure A_j , however, can not be consistently altered and should not be connected with a policy variable. Neither can the number of metro stations or the number of railway stations, as these variables are set in RETRO. For similar reasons, we can not connect policy variables to the area available for transport infrastructure. In the present version there are policy variables for the number of pupils (17-19 years old) and student in the schools and universities and also the area available for employment location I_{3i} , and the maximum number of workplaces per area unit in zone i WA_i (Table 4.2).

When y_{3i} is changed relative to the base scenario, it is important to be aware that the area for employment location plus the area for residential purposes and the area for other purposes must not exceed the total area of the zone.

Table 4.2. Policy variables available in the present version of EMP

Y_{2i}	Relative change in the maximum number of workplaces per area unit in zone i .
Y_{3i}	Relative change in the area available for employment location (km ²).
Y_{5i}	Relative change in the number of pupils in school (17-19 year).
Y_{6i}	Relative change in the number of students at Universities.

5 Data

For application of RETRO/IMREL input data are needed. In order to run the model for a specified time period; input data for this period must be available. Section 5.1, 5.2 and 5.3 describes the data that are necessary to run the RETRO/IMREL model and particularly a data set for the period around 1998. Data that can be used to estimate the modified RES and EMP sub models of IMREL are described in section 5.4. In order to apply the model for a future time period, input data for the future time period is needed. Section 5.5 explains methods to obtain input data for future years.

5.1 Input data to RETRO

The exogenous data that are needed to run the RETRO model includes data that are specific for the year that the model is applied for. This includes data for each of the 49 zones. The data describes population subgroup sizes, the total number of jobs, the number of jobs that attracts people (grocery stores, service and restaurants) and data for the total shopping centre area.

Total number of people in the zones are grouped according to age gender and employment status, where the age groups are 13-15, 16-24, 25-44, 45-64, 65-99. In this report, we will use data for the number of people in different subgroups divided by age and gender as of January 1, 1998, that are based on raw data from Oslostatistikken (1999) and Statistics Norway (1998). The data are adjusted according to the number of students that live in the Kringsjå area but are registered in other parts of the city or in counties in other parts of Norway (Prosam 65/97). Further subdivision into employed and unemployed, is based on the actual shares in a travel survey (Vibe, 1991).

Data for the total shopping centre area in the zones approximates the actual area in the end of 1998 and the beginning of 1999. Short-term and long-term parking costs are based on the travel survey by Vibe (1991). Data for the total number of workplaces in the zones W_{tot} and the share of workplaces w_i are dated November 1, 1997. The number of work places that attracts people in the zones S_i has the same date.

All the data were obtained from Oslo City planning authorities (PBE). The same data has also been used in the FREDRIK model for analysis of different alternatives for the Oslopakke 2 (SCC). The data are sufficient to establish an input file for application of the RETRO model for 1998.

5.2 Input data to RES

Residential location depends on the disutility, the share of the total number of workplaces in the region that are located in the different zones ($w_j = W_j/W_{tot}$), data that describe

transport costs between all pair of zones c_{ij} (NOK) and travel demand (P_{ij}) between all pair of the zones.

Data that are necessary as input to RES describe the disutility of living in the zone, transportation costs and travel demand and upper and lower limits on the proportions of the total population that can live in the different zones. Data for the transport costs and the travel demand are generated with the aid of the RETRO model. The share of workplaces in the zones are data that are common in RETRO and RES. Hence, the same data that are used with RETRO for calculation of the transport costs and travel demand can be used in RES. To describe disutility we use data for the area units of each city district that are regulated for housing supply in the zones in Oslo for the last half of 1999 which was obtained from PBE, whereas data for the area units in the municipalities of Akershus are current figures (2000) from Akershus County Municipality. The number of parking places is currently not obtained, and the technical problems for how to make EMME/2 calculate the total driving distance in the zones remains to be solved. As a tentative rule the upper limit for the number of people per area unit could be set 30 % higher than the number of people per area unit in 1998.

5.3 Input data to EMP

The input data to EMP are used to describe the attraction for employment location in the zones M_i and the maximum number of workplaces per area unit available for workplace location. The attraction measure consists of a measure for the accessibility to the work force plus several data that describe the attractions in the zone. The measure for accessibility to the work force includes data for the transport costs c_{ij} (NOK) between all pair of zones in the area and the share of housing supplies in the zones (h_i). We make the upper and lower limits on the number of workplaces dependent on both the area available for employment location and the maximum number of workplaces per area unit available for workplace location WA_{max} (section 4.3).

Data for the transport costs are generated with the aid of the RETRO model. The share of housing supplies must be consistent with corresponding input data to RETRO. RES can calculate the housing supplies. The density of metro stations and train stations in the city districts and municipalities that are valid for 1998 were obtained from timetables for the public transport operators and maps with city district borders and municipalities borders.

Data I_{3i} for the area units of each city district that are regulated for industrial purposes and workplaces in the zones in Oslo are for the last half of 1999 and was obtained from PBE. Corresponding data for municipalities in Akershus are data (for year 2000) from Akershus County Municipality. Data for the share of employees that have free parking I_{4i} , the number of pupils in school (13-16 year) I_{5i} , and (17-19) I_{6i} , the number of students at Universities I_{7i} , were obtained from PBE. The same data has also been used in the FREDRIK model for analysis of different alternatives for the Oslopakke 2 (SCC). The city authorities claim that the data are valid for 1998. The total areas with transport infrastructure I_{8i} are from the same data source as data for area available for industrial purposes and workplaces. As a tentative rule the value of WA_{max} could be set 30 % higher than the number of workplaces per area unit in 1998.

5.4 Data for calibration and parameter estimation

The RETRO model was developed and calibrated as part of previous work (Vold, 1999). Hence, the only thing to do to apply RETRO for 1998 is to run the model with new input data for 1998 until equilibrium is achieved. The transport costs and travel demand can then be calculated and used together with other exogenously given variables in RES and EMP.

For parameter estimation of RES and EMP, we need all the input data described in section 5.1, 5.2 and 5.3 and data for the endogenous model variables. For RES, the endogenous model variables are the share of households in the zones, and for EMP the endogenous variables are the share of workplaces in the zones. The input data are for the period around 1998. Thus for estimation we need additional data for household- and employment location for 1998 for the 49 zones (27 municipalities in Oslo and 22 counties in Akershus).

In order to obtain consistency with RETRO, the share of people in age groups from 13 years and above approximated data for the share of households in the zones. The data for the share of people in different subgroups divided by age and gender are valid for January 1, 1998, and are based on raw data from *Oslostatistikken* (1999) and Statistics Norway (1998). Data for the total number of workplaces in the zones W_{tot} and the share of workplaces w_i are dated November 1, 1997 (PBE). These data for residential- and employment location are the same that can be used as input to RETRO simulations when RES is not active (see section 5.1).

5.5 Data for model application for future years

For application of RETRO/IMREL for prediction purposes, we need data that describe the exogenous variables in the prediction year. Hence, application of the model for a future year requires that the year specific input data described in section 5.1, 5.2 and 5.3 be updated for the year that the model is applied for. Data for population forecasts are available from Statistics Norway and other sources. Forecasts for the number of workplaces are more difficult to obtain. Consequently, we have to use data for 1998. This does not have the implication that population growth lead to unemployment though. The only effect in the model is that the share of workplaces in the different zones is the same as in 1998. The same parking costs are used, but overall policy variables can be used to reduce the levels on parking cost to reflect the increasing income level. Data for future shopping centre area are also difficult to obtain.

Separate analysis can be made in order to construct input data for the total number of workplaces, workplaces that attracts people and shopping centre area. Future research in this respect would be to develop an economic growth model for the area that could endogenously calculate the number of workplaces in different industries and shopping centre area and to modify RETRO/ IMREL to handle several types of workplaces.

6 Model implementation and solution algorithm

The data flows from the RETRO part to the IMREL part of RETRO/IMREL consists of overall travel costs c_{ij} and travel demand P_{ij} . The travel costs are represented as the logsum of travel costs for the modes and travel demand is represented in terms of shares of total travel demand. Feedback to RETRO yields housing supply and work places in the zones (Figure 6.1).

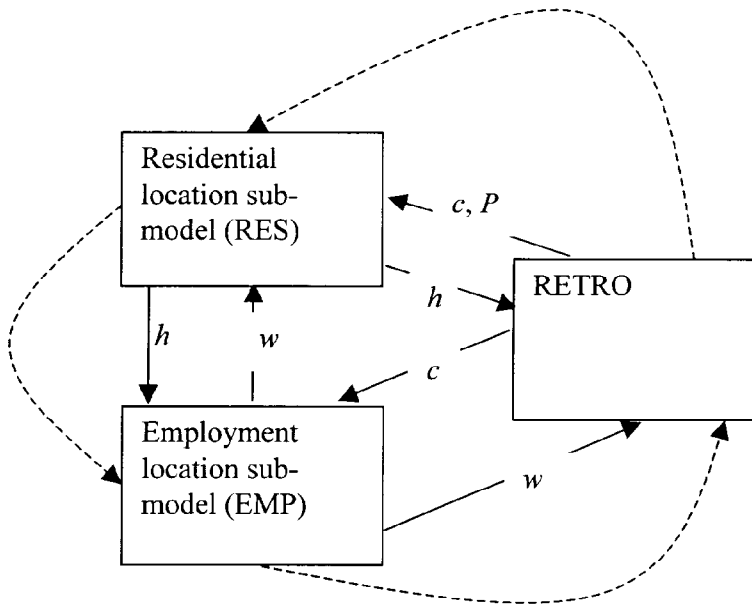


Figure 6.1. Schematic view of the data flow in RETRO/IMREL, and the iterative three-stage solution algorithm. Solid arrows show data flow and dashed lines show the sequential evaluation of the sub-models.

When the complete model system is run, the sub models are run in loop and simultaneously interchanging data are. RETRO produce input data of transport costs and travel demand to RES and EMP. EMP produces data for the share of workplaces in the zones that are used as input to RETRO and RES. Finally, RES produce data for the share of households in the zones that is used as input to RETRO and EMP. The RETRO model use data for residential location and employment location from the respective sub models, EMP use data of residential location from RES and some exogenous data from RETRO,

whereas RES use data of employment location and some exogenous data from RETRO. Newly calculated values for employment and residential location is then used as input to RETRO and so forth until hopefully convergence is obtained, the loop is terminated and equilibrium is achieved.

6.1 Solution algorithm

We use an iterative multi-stage solution algorithm to solve the optimisation problem that is solved while running RETRO/IMREL. This algorithm differs slightly from the algorithm given by Boyce and Mattsson (1999). It is a heuristic procedure of successive approximation that is described as follows (see also section 6.4 in Vold, 1999):

Set $k = 0$ and choose a feasible housing supply vector h_{ij}^k , an employment location vector w_{ij}^k , and an OD matrix for the share of trips between zones P_{ij}^k .

- 1 Use RETRO to calculate User Optimal travel costs $c_{ij}(h^{k+1})$ using fixed demand P_{ij}^k , and h^k , and w^k .
- 2 Run EMP, given h^k and $c_{ij}(h^{k+1})$, yielding the employment location, w^{k+1} .
- 3 Run RES to calculate h^{k+1} , given $c_{ij}(h^{k+1})$, P_{ij}^k and w^{k+1} .
- 4 Use RETRO to calculate travel demand \tilde{P}_{ij}^{k+1} based on $c_{ij}(h^{k+1})$, h^k , and w^{k+1} . If $\max_{ij}(|\tilde{P}_{ij}^{k+1} - P_{ij}^k|) < \varepsilon$, then stop. Otherwise set $P_{ij}^{k+1} = P_{ij}^k + \alpha \cdot (\tilde{P}_{ij}^{k+1} - P_{ij}^k)$, where $\alpha = \frac{1}{k+1}$ guarantee convergence, then set $k = k + 1$ and goto 1

When convergence is obtained, further loops do not change the results from any of the three sub models in the loop. Hence this solution is an equilibrium solution. We can not be sure that it is the only equilibrium solution, however. Other equilibrium solutions may exist, where the value of the objective functions of the sub models becomes even higher.

This implies that we can not be sure that the base case alternative represents the only equilibrium solution. Other starting values could result in a different equilibrium solution. Furthermore, we do not know which of perhaps several possible equilibrium solutions we obtain by running the model for an alternative scenario.

Results from base case scenarios and alternative scenarios are often used in comparative analysis. There is a danger that the results from the base scenario and the alternative scenario can not be consistently compared. We have not experienced problems of this kind while applying RETRO and RETRO/IMREL, however, and it is believed that a critical eye would uncover such cases. Although it is unlikely that we will experience situations where there are several equilibrium solutions and where this result in inconsistent comparison of scenarios, it is important to be aware that this could potentially be a problem.

6.2 Model implementation

The combined RETRO and IMREL model was implemented in a C-program according to the solution algorithm described in the previous section.

For model implementation of RES and EMP we used the C-version of a constrained optimisation algorithm DONLP2 (7/99) developed by P. Spellucci (1999), Technical University at Darmstadt. Initial values of travel costs; employment location and the number of trips between zones are read from input files.

A function for each of the four stages in the solution algorithm was build and evaluated sequentially in the C-program. Writing to data files and reading from data files makes data transmission between the sub-models.

The complete model system is currently installed on a machine running the UNIX HP-UX 9000 operative system with a D270 CPU.

6.3 File structure

The sub-models of RETRO/IMREL interchange data by reading to and writing from data files. Files that are used by the RETRO model are described in Vold (1999). For convenience, however, the RETRO model was modified such that results are written to two extra data files. The extra data file named *sum_cost.txt* contains lumped travel costs within and between zones represented as

$$(6.1) \quad c_{ij}' = \sum_{m \in \mathbf{N}_{nd}} e^{(\tilde{v}_m^{ij} + \tilde{v}_{md}^{ij}) \cdot \mu_m}$$

Hence the logsum of equation (4.1) can be expressed as

$$c = \frac{1}{\mu_m} \ln(c').$$

The other datafile named *travel_share.txt* contains data for the shares of the total number of trips within and between zones.

All sub-models require an input file with initial data that characterises the zones. Among the data in this file are the numbers of people that live in the respective zones and the number of people that work in the respective zones. RES and EMP recalculate the number of people that live in the different zones and the number of people that work in the different zones, respectively. Both models write the recalculated values to an updated copy of the input file. If the year is 1995, the updated copy is named *IMREL1995.txt*.

When the sub-models are run interchangeably in a loop, the input file with initial data is used as input to RETRO and the housing location sub-model only in the first iteration. The housing location sub-model writes the updated data to *IMREL1995.txt*, which is used as input to the employment location sub-model. In subsequent iterations *IMREL1995.txt* is both used as the input file to all the sub-models, and updated by the housing location and employment location sub-models.

7 Estimation of the RETRO/IMREL model

RETRO/IMREL consists of sub models that must be estimated before they can be applied. RETRO has been calibrated and estimated as part of previous work (Vold, 1999). This Chapter deals with parameter estimation in RES and EMP.

RES is estimated with respect to data for the share of households in the zones $h_i, i=1, \dots, n$ and input data for the period around 1998 as described in Chapter 5. EMP is estimated with respect to data for the share of work places in the zones and input data.

While calibrating and estimating RES with respect to data from the period around 1998, input data includes the calculated transport costs and travel demand from RETRO for the actual situation in 1998 and the data for workplace location that are valid for the same period.

Similarly while parameters are calibrated and estimated in EMP, input data includes the calculated transport costs and travel demand from RETRO for the actual situation in 1998 and the data for residential location that are valid for the same period.

When RETRO, RES and EMP are run in loop, RETRO get calculated values of residential location and employment location from RES and EMP. When data for residential- and employment location are available, however, these data can be used as input to RETRO instead. RETRO with the same data for the proportions of households and workplaces in the zones that were used for estimation in RES and EMP plus other data that describe the situation around 1998 is used to calculate transport demand and transport costs between zones for the period around 1998.

This means that there is no need for iterated interactions between the sub models while parameters are estimated. Hence, the sub models can be estimated separately as the data otherwise obtained from other models are given as empirical data or from pre-calculated input data from RETRO.

7.1 Estimation of RES

In section 4.2 it was decided to reduce the disutility model of equation (4.4) to

$$d_i = \zeta_1 \cdot DP_i(h_i, TH, x_{1i} \cdot RH_i)$$

The part of housing location in the zones that is not explained by the disutility function and transport costs can be explained by the zone specific parameters. Thus, the parameters in RES that must be estimated are μ and $\phi_i, i = 1, \dots, n$ and ζ_1 . Additionally upper and lower limits of h_i must be set.

Grue, Langeland and Larsen (1997) used two data sets to assess the impact of road traffic noise, the number of housing units per 1000 m² and other variables on market price of housing properties. The first data set comprises selected flats and houses belonging to the co-operative housing society (OBOS) and includes 4638 units that were sold in 1995. The second data set includes owner occupied flats and owner occupied single family, semidetached and terraced houses and includes 10014 units that were sold in the period 1988-1995. A hedonic price approach was used and the hedonic prices were estimated for each of the variables by standard regression technique. Their estimates on the impacts were significant for all the variables that were part of the analysis. Their results indicates that prices decrease by 0.5 % per dBA for owner occupied housing, whereas for apartments in the housing co-operative the estimated impact is only half this value. They explain this discrepancy in that the noise coefficient also captures other disamenities caused by road traffic and that these – on the average – are less severe for apartments belonging to the housing co-operative. The data set for the co-operative housing included variables that made it possible to estimate that the impacts of one extra housing unit per 1000 m² on housing prices is -3.25 %. The other data set did not include data for estimating this kind of impact. In the present version of RETRO/IMREL, we will assume that this estimate is valid for any housing types. Hence, we could set $\hat{\xi}_1 = 0.0325$. As this figure is based on apartments in OBOS it is of course not guaranteed that this parameter has a similar value for all types of residential areas. In lack of other estimates, we may stick to this value for all residential areas.

In the present version of RETRO/IMREL we decided, however, not to use this estimate. We found it more convenient to estimate all parameters simultaneously. To do this we used the dual formulation of the mathematical programming formulation (4.3) of RES:

$$\min_{\lambda, \mu} \phi(\lambda, \mu) = \min_{\lambda, \mu} [\max_{\mathbf{h}} (V(\mathbf{h}) + \lambda \cdot (\sum_i h_i - 1) + \mu' \cdot \mathbf{g}(\mathbf{h}))] ,$$

where $\lambda, \mu \geq 0$ and $\mathbf{g}(\mathbf{h})' = [h_1 - u_1, h_2 - u_2, \dots, l_1 - h_1, l_2 - h_2, \dots] \leq 0$.

To obtain the values for the zone specific parameters that explains the part of the housing location that is not explained by the disutility function, they must be calibrated such that the model calculate housing shares h_i that are equal to the actual housing share for the base year \tilde{h}_i .

We wanted to find parameter estimates for which the model predicts the actual housing location for the period around 1998. Housing shares for the base year \tilde{h}_i are within the actual upper and lower bound for housing location and $\sum_i \tilde{h}_i = 1$, which implies that we can set $\lambda, \mu = 0$. Hence, in order to make the model reproduce the actual housing shares for the base year, we have that the parameters must be set such that $\nabla_{\mathbf{h}} V(\tilde{\mathbf{h}}) = 0 \forall i$.

The zone specific parameters ϕ_i must be set such that

$$(7.1) \quad \nabla_{h_i} V(\tilde{\mathbf{h}}) = -h_i \cdot \frac{d(d(h_i))}{d(h_i)} - d_i(h_i) - \sum_j (P_{ij} \cdot c_{ij}) + \phi_i = 0 \quad \forall i$$

As an alternative to estimating only the zone specific parameter ϕ_i , we additionally consider μ and ξ_1 to be unknown. To estimate μ , ξ_1 and $\phi_i \forall i$ we will use the method of maximum likelihood, where an OD matrix for the total travel demand between zones is used as data. We use the OD matrix obtained while running the RETRO model for the base year. But to obtain an OD matrix for the travel demand between the zones N_{ij} , that is consistent with \tilde{h}_i , we let $N_{ij} = \frac{\tilde{h}_i}{P_i} \cdot P_{ij}$ where $P_i = \sum_j P_{ij}$.

The maximum likelihood method for estimation of parameters in RES, maximises the likelihood function

$$L = \prod_{i,j} \left[\frac{w_j \cdot e^{-\mu(q_i + d(h_i) + c_{ij} + \phi_i)}}{\sum_i e^{-\mu(q_i + d(h_i) + c_{ij} + \phi_i)}} \right]^{N_{ij}},$$

with respect to μ , ξ_1 and $\phi_i \forall i$ and simultaneously makes sure that $\nabla_h V(\tilde{\mathbf{h}}) = 0 \forall i$. The latter is obtained if $V(\tilde{\mathbf{h}}) = \max V(\mathbf{h})$. Hence, we may formulate the method as the following two level optimisation problem:

$$1.\text{level:} \quad \max_{\mu, \xi_1} L(\mu, \xi_1, \phi_1, \dots, \phi_n)$$

where ϕ_1, \dots, ϕ_n solve

$$2.\text{level} \quad \max_{\phi_1, \dots, \phi_n} V(\mathbf{h}) = V(\tilde{\mathbf{h}})$$

If we express q_i in terms of equation (4.2) the likelihood function can be expressed by

$$L = \prod_{i,j} \left[\frac{w_j \cdot e^{-\mu(-\sum_k (P_{ik} \cdot c_{ik}) + c_{ij} + \phi_i)}}{\sum_i e^{-\mu(-\sum_k (P_{ik} \cdot c_{ik}) + c_{ij} + \phi_i)}} \right]^{N_{ij}}$$

Moreover, a solution to the lower optimisation problem must satisfy

$$\nabla_h V(\tilde{\mathbf{h}}) = -h_i \cdot \frac{d(d(\tilde{h}_i))}{d(\tilde{h}_i)} - d(\tilde{h}_i) - \sum_j (P_{ij} \cdot c_{ij}) + \phi_i = 0 \quad \forall i.$$

Thus,

$$(7.2) \quad \phi_i = \tilde{h}_i \cdot \frac{d(d(\tilde{h}_i))}{d(\tilde{h}_i)} + d(\tilde{h}_i) + \sum_j (P_{ij} \cdot c_{ij}) \quad \forall i$$

By using this expression for ϕ_i in the likelihood function, we have reduced the two level optimisation problem to an ordinary optimisation problem of maximising the likelihood function with respect to μ , ξ_1

$$L = \prod_{i,j} \left[\frac{w_j \cdot e^{-\mu \left(\frac{d(\tilde{h}_i)}{d\tilde{h}_i} \cdot \tilde{h}_i + d(\tilde{h}_i) + c_{ij} \right)}}{\sum_i e^{-\mu \left(\frac{d(\tilde{h}_i)}{d\tilde{h}_i} \cdot \tilde{h}_i + d(\tilde{h}_i) + c_{ij} \right)}} \right]^{N_{ij}}$$

Estimates for μ , ξ_1 can be used in equation (7.2) to calculate estimates for ϕ_i . The log likelihood function is given by

$$\text{Log}(L) = \prod_{i,j} N_{ij} \cdot \left[\text{Log}(w_j) - \text{Log} \left(\frac{e^{-\mu \left(\frac{d(\tilde{h}_i)}{d\tilde{h}_i} \cdot \tilde{h}_i + d(\tilde{h}_i) + c_{ij} \right)}}{\sum_i e^{-\mu \left(\frac{d(\tilde{h}_i)}{d\tilde{h}_i} \cdot \tilde{h}_i + d(\tilde{h}_i) + c_{ij} \right)}} \right) \right]$$

Estimates obtained by using the Nelder-Mead simplex algorithm for optimisation of nonlinear functions (see Appendix) to solve the optimisation problem

$$\max_{\mu, \xi_1} \text{Log}(L(\mu, \xi_1))$$

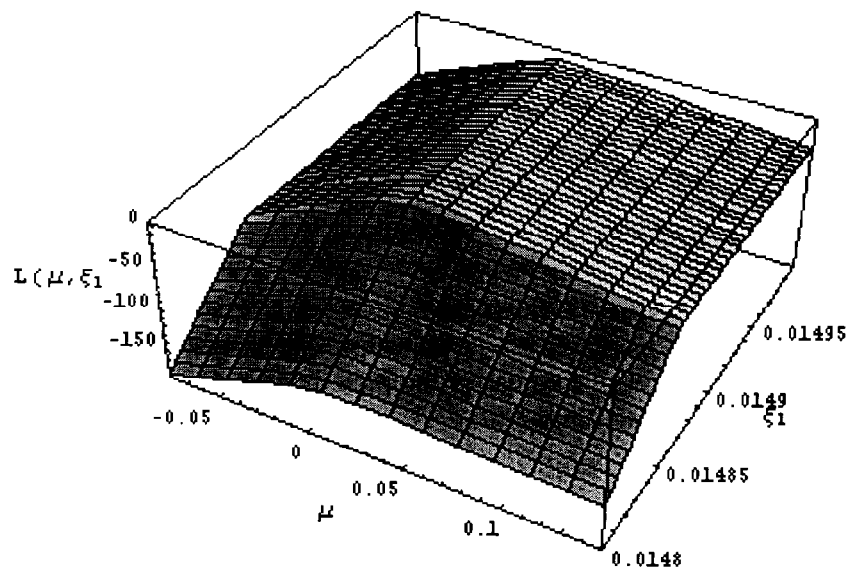
gives the estimates $\hat{\mu} = 0.012020$ $\hat{\xi}_1 = 1.230258$ (Figure 7.1). Application of equation (7.2) with these estimates gives estimates of the zone specific constants $\hat{\phi}_i \forall i$. The estimate of the logsum parameter $\hat{\mu}_m = -0.0235$ and other parameters in the logsum for transport costs were calculated on the basis of Vold (1999). Hence the relationship between $\hat{\mu}$ and $\hat{\mu}_m$ is theoretically sound (see Ben-Akiva, 1985). We have used a two stage algorithm in the sense that parameters for transport costs in the logsum and the logsum parameter was estimated first (Vold, 1999). There these estimates were used in the model where the rest of the parameters were estimated. In Ben-Akiva (1985) this way of estimating the model is denoted the sequential procedure.

Table 7.1. Zone specific parameter estimates $\hat{\phi}_i$, $\forall i$

Oslo		Akershus	
Bygdøy Frogner	-1.332363	Vestby	1.026999
Uranienb. Majorst.	-2.558763	Ski	-0.098953
St.Haug Ullevål	-2.734429	Ås	0.447117
Sagene Torshov	-2.345481	Frogn	1.079196
Grunerløkka. Sofi.	-2.219445	Nesodden	0.987641
Gamle Oslo	-1.575345	Oppegård	-0.254382
Ekeberg Bekkelag.	-0.297724	Bærum	-12.307756
Nordstrand	-0.884414	Asker	-2.877375
Søndre Nordstrand	-1.128266	Aurskog-Høland	1.456984
Lambertseter	-0.348560	Sørums	0.684394
Bøler	-0.643191	Fet	0.267109
Manglerud	-0.737947	Rælingen	-0.354802
Østensjø	-1.055742	Enebakk	0.820169

Table 7.1. Zone specific parameter estimates $\hat{\phi}_i, \forall i$ continues

Helsfyr Sinsen	-1.968848	Skedsmo	-0.092626
Hellerud	-0.984346	Lørenskog	-2.081172
Furuset	-1.946215	Nittedal	0.260607
Stovner	-1.166995	Gjerdrum	0.277009
Romsås	-0.427200	Ullensaker	1.793754
Grorud	-0.953606	Nes	1.642798
Bjerke	-1.783846	Eidsvoll	1.187791
Grefsen Kjelsås	-1.433455	Nannestad	0.763683
Sogn	-1.906039	Hurdal	0.255423
Vinderen	-1.337013		
Røa	-1.662538		
Ullem	-2.098214		
Sentrum	-0.133582		
Marka	-0.039222		

**Figure 7.1.** Log-likelihood function $L(\mu, \xi_1)$

7.2 Estimation of EMP

The parameters that must be estimated in EMP are the parameters in the employment location function $M_j(\gamma \cdot \alpha, \gamma \cdot \beta_1, \dots, \gamma \cdot \beta_n)$. We may use data of the number of workplaces in the zones in a base year for estimation of the model parameters $\gamma \cdot \alpha$ and $\gamma \cdot \beta_k, k = 1, \dots, n$. Solutions of EMP (equation 4.5) satisfies the first order conditions

$$\nabla_{w_j} f(\mathbf{w}) = \frac{\ln(w_j)}{1} + 1 - 1 - M_j = \ln(w_j) - M_j = 0$$

Hence, in order to ensure that the model gives the solution

$$f(\tilde{\mathbf{w}}) = \min_{\mathbf{w}} f(\mathbf{w})$$

for our data from the period around 1998, we must have that

$$\ln(\tilde{w}_j) = \alpha \cdot A_j + \sum_k \beta_k \cdot I_{k1} + \hat{\varepsilon}_j,$$

where $\hat{\varepsilon}_j$ is a zone specific parameter, and \tilde{w}_j are the data points. Ben-Akiva (1985, p.120) explain how the parameters can be estimated by regression analysis. What we actually may do is to first consider $\hat{\varepsilon}_j$ as residuals. Then, the parameters $\gamma \cdot \alpha$ and $\gamma \cdot \beta_k, k = 1, \dots, n$ are easily estimated by linear regression. The residuals from linear regression can be used as zone specific parameters.

We applied the algorithm for linear regression of Mathematica 3.0 Standard Add-on packages. The results include the Partial Sum of Squares, which gives a partitioning of the model sum of squares. The partial sum of squares consists of one element for each non-constant basis function included in the model. The elements show how much the elements contribute to the explanatory power of the model. The Coefficient of determination R^2 was 0.69. Only four parameters were significant: $\gamma \cdot \hat{\alpha}$, $\gamma \cdot \hat{\beta}_5$, $\gamma \cdot \hat{\beta}_6$ and $\gamma \cdot \hat{\beta}_8$. The estimates of these parameters did also have the largest Partial Sum of Squares. Hence we decided to omit all other components. We redid the regression with these parameters only (Table 7.2 and 7.3). The new coefficient of determination was 0.675, and the partial sums of squares were 12.97, 4.23, 2.10 and 7.43, respectively, indicating that the accessibility measure and the area available for industrial purposes are of greatest importance for location of workplaces. In order to make the model fit data for the period around 1998, the residuals were used as zone specific parameters (Table 7.4)

Table 7.2. Parameter estimates for the employment location sub model

Variable	Estimate	SE	TStat	PValue
1	-6.321	0.249	-25.408	0
Accessibility, $\gamma \cdot \hat{\alpha}$	0.557	0.107	5.208	4.84e-6
High school, $\gamma \cdot \hat{\beta}_5$	0.000410	0.000138	2.976	0.00474
University, $\gamma \cdot \hat{\beta}_6$	3.95e-5	1.89e-5	2.096	0.0419
Work area, $\gamma \cdot \hat{\beta}_8$	0.000441	0.000112	3.942	0.000286

Table 7.3. Anova table for EMP

	Degrees of freedom	Sum of squares	Mean squares	F-ratio	P-value
Model	4	43.657	10.914	22.830	0
Error	44	21.035	0.478		
Total	48	64.692			

Table 7.4. Estimates of zone specific parameters $\hat{\varepsilon}_j$

Oslo		Akershus	
Bygdøy Frogner	-0.246	Vestby	-0.356
Uranienb. Majorst.	0.831	Ski	0.256
St.Haug Ullevål	-0.498	Ås	0.146
Sagene Torshov	0.814	Frogn	0.586
Grunerløkka. Sofi.	0.596	Nesodden	0.062
Gamle Oslo	1.196	Oppegård	0.411
Ekeberg Bekkelag.	-0.177	Bærum	-0.510
Nordstrand	-0.621	Asker	0.052
Søndre Nordstrand	0.142	Aurskog-Høland	0.093
Lambertseter	-0.548	Sorrum	-0.011
Bøler	-0.193	Fet	-0.566
Manglerud	-0.314	Rælingen	-0.780
Østensjø	-0.470	Enebakk	0.279
Helsfyr Sinsen	0.114	Skedsmo	-0.252
Hellerud	0.102	Lørenskog	0.057
Furuset	0.440	Nittedal	-0.008
Stovner	-0.063	Gjerdrum	-0.507
Romsås	-1.455	Ullensaker	-0.938
Grorud	0.194	Nes	-0.356
Bjerke	0.138	Eidsvoll	0.459
Grefsen Kjelsås	0.340	Nannestad	0.085
Sogn	-0.260	Hurdal	0.314
Vinderen	-0.175		
Røa	-0.009		
Ullern	1.120		
Sentrum	2.313		
Marka	-1.893		

8 Model application

The description of model equations and model estimation for RETRO/IMREL as outlined above defines a combined land-use and transport model for the greater Oslo area. The next step is verification of the model.

In this Chapter a simple verification of RETRO/IMREL is described. This includes description of assumptions for test runs and some qualitative and quantitative results.

8.1 Qualitative results

The household's choice of residential location depends on the disutility $d_i, i = 1, \dots, n$, of living in the zones, on the travel costs to the work places, $c_{ij}, i, j = 1, \dots, n$, in other zones, and the share of travel between pairs of zones P_{ij} .

To investigate qualitative characteristics of RES, we let $P_{ij} = 1/(n \cdot n)$, which means that we assume there are the same number of trips between all pair of zones, and we set all upper limits for h_i at 0.9 and all lower limits at 0.01.

First we set $\xi_1 = 0.0$, which means that disutility counts nothing in the model. With this disutility function, the model moves a maximal amount of household to the Helsefyr-Sinsen zone. The number of households in the rest of the zones is at the lower limit. The reason for the concentration in Helsefyr-Sinsen is because overall travel costs to other zones in the region are lowest for this zone.

Employment location depends on the accessibility to labour force and other attraction measures that makes a zone attractive for employment location expressed in terms of M_i . If we set γ close to zero, differences between zones are diminished. As a consequence the model establish equal amounts of work places in all zones. On the other hand, if γ is set at a large value the upper limit for employment is reached for the zone where the attraction measure is largest. If γ is set in-between, then we can obtain a balance between spread of employment and concentration in attractive zones.

It is noted, however, that the appropriate value of γ depends on the values of parameters in the accessibility measure (equation 4.6), which are temporarily set more or less arbitrary.

8.2 Quantitative results

We will investigate the long-term effects in (1) a scenario where the fuel tax is tripled and in (2) a scenario where the toll charge is quadrupled relative to a base scenario with

housing location and employment location for the period around 1998. The effects we will consider are:

- a) Altered shares of the total number of household locations in the zones.
- b) Altered shares of the total number of workplaces that are located in the zones.
- c) Changes in the total distance by car in the region.
- d) Changes in the number of trips by car, public transport and walk/bicycle.

As prices change, it takes time before household- and workplace location again is in equilibrium. RETRO/IMREL calculates the long-term effects of changed prices. This means that the results are for a future year, where household- and employment locations have reached a new equilibrium. We can not say how long time it will take before the new equilibrium is reached. The time horizon depends on how narrow we set the intervals that are bounded by the upper and lower bounds on shares of residential - and employment location and the size of the areas available for different purposes. In real life it seems reasonable that at least 10 years is needed to capture a significant trend.

For the test cases, we assume no growth in the total population and no changes in the total number of workplaces. The upper and lower limits on the share of housing supply h_i in the zones were set at 0.9 and 0.00001. The same limits were used for the share of workplaces w_j in the zones $i, j = 1, \dots, n$.

In the scenario with a tripled fuel tax, we see that the number of both residences and workplaces increase in areas close to the city centre (Figure 8.1). However, there is an insignificant increase in the number of residences in the city centre, where the area for residential purposes is small and the number of residences is small in the base scenario (ca. 1000). The city centre contains 87 000 workplaces in the base scenario. This increases by about 30 %.

Also in the scenario where toll charges are quadrupled, we obtain a centralisation in areas that are close to the city centre (Figure 2). Different from the scenario with an increase in the fuel price is that we also get migration of households and work places to Asker municipality some distance south west of Oslo.

In the scenario with an increase in the fuel price it was of importance to reduce the fuel expenses. In the scenario with increasing toll ring charges, we have that households and employees tend to localise in a way that reduces the number of trips through the toll ring. Because more households and work places inside the toll ring increases the disutility in these zones, a share chooses instead to establish in Asker in which there are many residences and workplaces initially in the base scenario.

The transport data for the three scenarios (Table 8.1) show that increased fuel tax gives a greater effect on the mode choice and total distance by car than changes in the toll charge. In the case with increased toll charges and in the case with increased fuel tax, the travellers may reduce the expenses by relocating households and workplaces. In the scenario with an increase in the fuel price, the car ownership is affected through the car ownership model. As it is not possible to put toll charges in the car ownership model, we have that car ownership was not changed in the scenario with an increase in the toll charge. Transport data is affected by changes in the car ownership, but we will not in this example calculate how much that is caused by changes in the car ownership.

Based on the car distances of the base scenario and the scenario with tripled fuel tax, the elasticity of total car distance in the peak period with respect to the fuel tax becomes -0.15 .

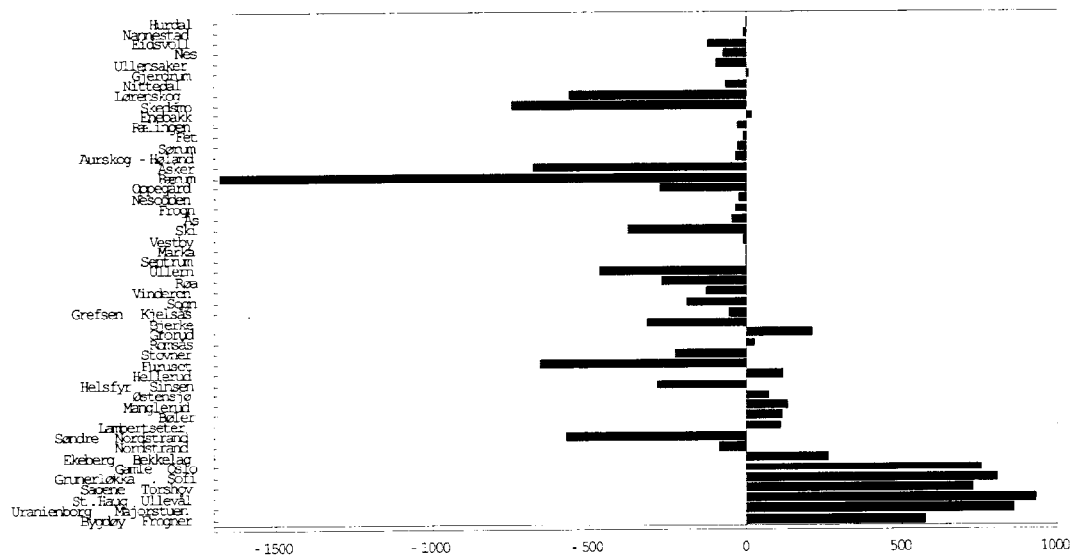


Figure 8.1. The effect of tripled fuel tax on household relocation

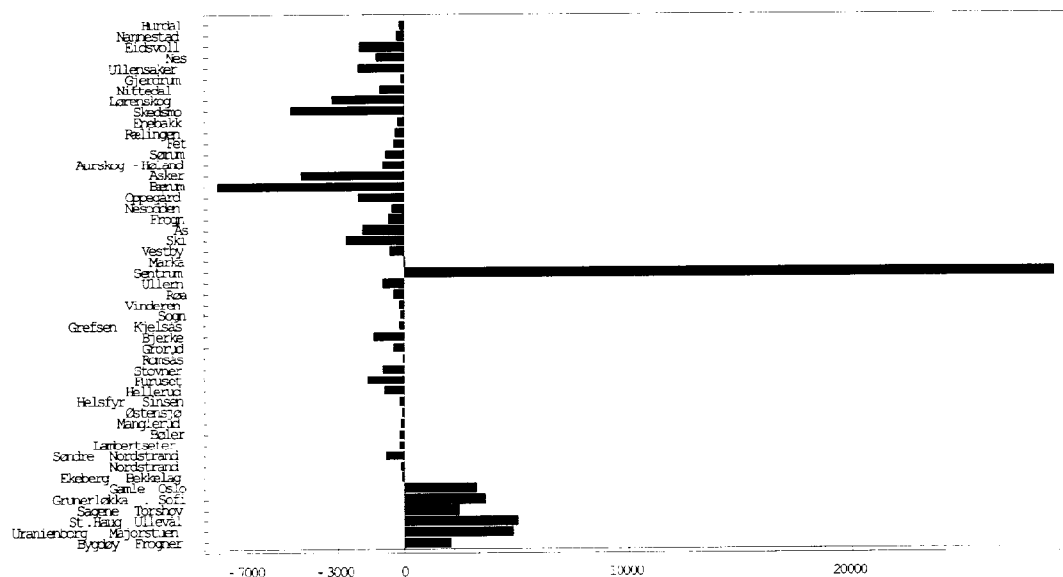


Figure 8.2. The effect of tripled fuel tax on employment relocation

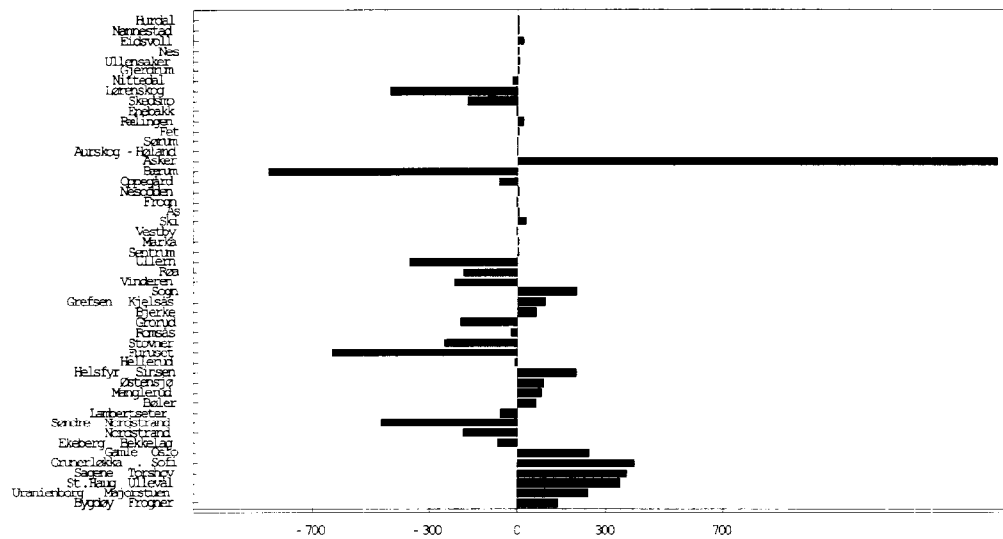


Figure 8.3. The effect of quadrupled toll ring charge on household relocation

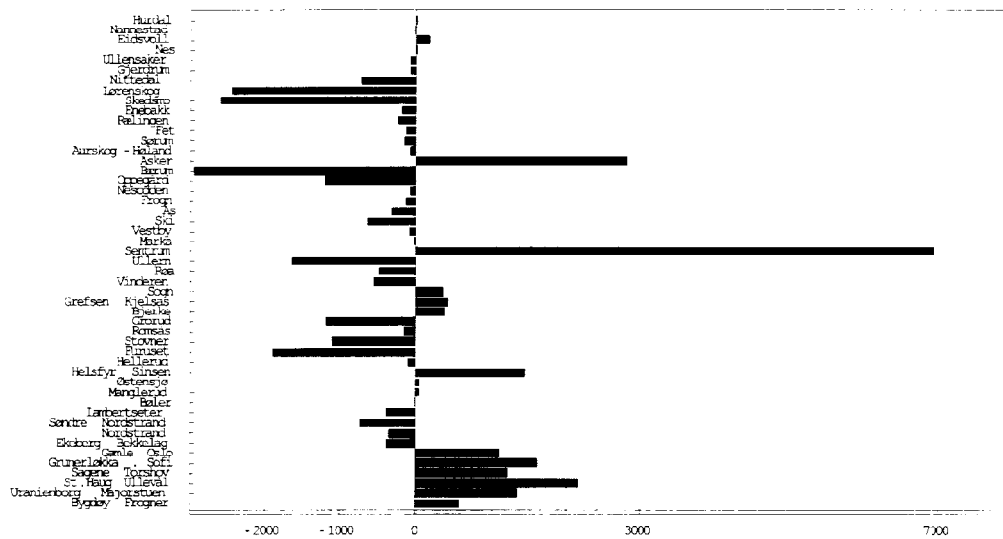


Figure 8.4. The effect of quadrupled toll ring charge on employment relocation

Table 8.1. Aggregate travel demand and total car distance during the peak period (4.35 hours) in the scenarios

	Base case (1998)	Tripled fuel tax	Quadrupled toll ring charge
Car trips	446835	342228	431882
Public transport trips	265026	328949	273390
Walk/bicycle trips	107623	146935	114025
Car distance (km)	9075342	7688612	8784068

9 Conclusions

This report describes RETRO/IMREL - a combined land-use and transport model for the greater Oslo area and applications on simple test cases. Equilibrium models for transport- and land use planning like RETRO/IMREL are not expected to be capable to assess a very precise forecast for the amount of relocation among the zones relative to a base case. It is expected, however, that results from models of this type are reasonable and that the trends are reliable.

The intervals defined in terms on boundaries on the shares of households and workplaces in the zones are very wide (0.00001-0.9) in the test scenarios we have analysed. These intervals will be much narrower in realistic scenarios. The boundaries have great influence on the final result and must be carefully set. The values on the boundaries depend on the areas available for different purposes in the zones and the time horizon of the actual scenario. In the short run only a fraction of the households and employees are able to change location. More people will be able to relocation in the long run than in the short run. Thus to avoid that an unrealistic amount of households and employees relocate, not only must we take into account that the areas available for different purposes are limited, we must also take into account that the time horizon of the actual scenario do put constraints on the amount of relocation.

In conclusion, the qualitative and quantitative model behaviour of RETRO/IMREL seems reasonable. The cases were simple, however, so further development, validation and use is required in order to obtain more interesting results.

For future work, it would be interesting to include exogenously given variables in the disutility function. Interesting in this respect would be to include a variable that represents the service level, the availability for parking space, external environmental costs and a property tax. New variables for description of the establishment of workplaces would also be interesting. For this purpose it would be interesting to include an endogenously calculated variable that express the concentration of workplaces or the closeness to other workplaces. For future work it is also a challenge to develop a system that make it possible to use RETRO/IMREL in cost-benefit analysis of land use and transport policy instruments.

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Appendix

Appendix

The log likelihood can be expressed as a function $f(\mathbf{p}) = \ln(L(\mathbf{p})) : R_m \rightarrow R$ with respect to a vector of n variables $\mathbf{p} = [p_1, \dots, p_m]$. A non-linear optimisation algorithm is needed in order to obtain a solution to the maximisation problem,

$$\max_{\mathbf{p} \in R_m} f(\mathbf{p})$$

Non-linear optimisation algorithms are based on different principles. An important class of algorithms require the derivative, $df/d\mathbf{p}$, whereas so-called DUD algorithms (Doesn't Use Derivatives) does not. In general the former have higher order rates of convergence, whereas the latter are more robust and easy to apply.

Optimization algorithms

The choice of optimisation algorithm for maximisation of a given objective function depends on certain qualities of the objective function.

Some optimisation algorithms require the value of the derivative of the objective function for arbitrary values of function arguments. The derivatives of simple functions can often be expressed as analytical functions.

For other functions, finite differences can be used to approximate the derivatives. Although algorithms that use values of the derivative is often efficient in terms of function evaluations, it is sometimes cumbersome to establish the routine that calculates the values of the derivatives.

Some optimisation algorithms apply one-dimensional minimisation along lines in multidimensional space. Powell's Method (Press et al, 1988) is of this type. It uses information about the optima obtained from previous line maximisation in the multidimensional space in order to choose new directions for line maximisation.

A line maximisation algorithm makes the line maximisation. Hence, to apply optimisation algorithms of this type, it is necessary to set two convergence criteria - one for line maximisation and one for the overall multidimensional function maximisation.

Application of the Simplex algorithm requires the specification of only a single convergence criterion. Evaluation of the derivative of the objective function is not needed. Besides this, the method is easily understood and implemented. However, it is not very efficient in terms of the number of function evaluations it requires.

Values of the derivative of W are not easily available. Moreover, it is not easy to pre-set the convergence criterion. Due to the long computing time, it is more desirable to monitor the change of function and parameter values and terminate the optimisation algorithm when changes are considered small. Considering the long computing time, this gives a flexible way of compromising between the number of iterative runs and the accuracy of the calculated optimum.

Hence, the Simplex algorithm seems like a good starting point in the attempt to understand how to optimise the W function. Experience with the Simplex method may be of great value if more sophisticated algorithms are introduced at a later occasion.

The Simplex method is well suited for optimisation of W with and without max-min constraints on independent variables (policy measures).

Simplex method

The Downhill Simplex Method is a very robust and easy to use DUD method in Multidimensions (Nelder and Mead, 1965; Press et al. 1988). A simplex is the geometrical figure consisting, in m dimensions, of $m+1$ points (or vertices) and all their interconnecting line segments. In two dimensions, a simplex is a triangle, and a tetrahedron in three dimensions. Initially, the algorithm choose the vertices at $m+1$ points $\mathbf{P}=\{\mathbf{p}_0, \mathbf{p}_1, \dots, \mathbf{p}_m\}$ that span R_m . The points can be given by the formula

$$\mathbf{p}_i = \mathbf{p}_0 + [0, \dots, \lambda_i, \dots, 0], \quad i = 1, \dots, m$$

where \mathbf{p}_0 is some the initial guess and λ_i is a scalar. The function, f , is evaluated at each of the vertices. The vertices are then moved towards the maximum of, f . In each iteration, k , one out of four formulas,

$$\mathbf{P}^{k+1} = g_i(\mathbf{P}^k), \quad i = 1, \dots, 4$$

is used to find the new position of the points \mathbf{P} . The moves are made according to certain rules, which ensure that the simplex is never degenerated. The algorithm moves 1 or m points per iteration. Characteristics of the algorithm is that the simplex expands where $f(\mathbf{p})$ is smooth and increasing, and contracts close to the maximum and where the function surface is rugged. Figure 5.1 to 5.7 shows the possible moving of vertices in iterations.

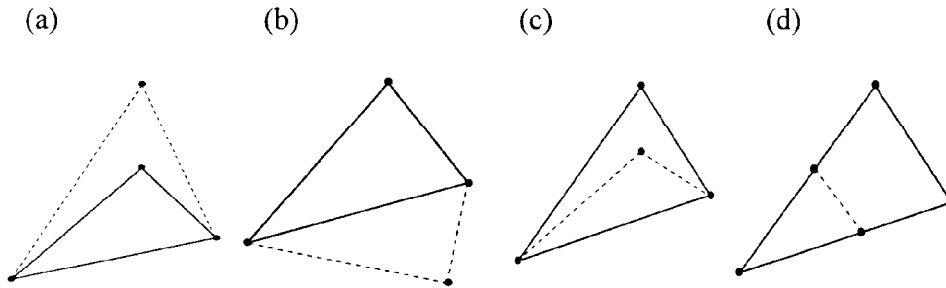


Figure A1. The results of formulas g_i , $i=1, \dots, 4$ that are used to move vertices of the simplex. The moves are (a) expansion, (b) mirroring, (c) truncating and (d) shrinkage, respectively.

Reparametrization

Elements p_i , $i=1, \dots, m$ are economically interpretable and constrained between a lower and an upper limit, $p^{(l)} \leq p \leq p^{(u)}$. Unconstrained optimisation with respect to \mathbf{p} may give meaningless estimates $\hat{\mathbf{p}}$ that are beyond the limits. However, transformation of the

parameters (policy measures) with the reparametrisation by Vold et al. 1999, $\xi(p) = \log((p - p^{(l)})/(p^{(u)} - p))$, ensures that an original parameter p stays within its boundaries during unconstrained estimation. Since $e^{\xi} = (p - p^{(l)})/(p^{(u)} - p)$, which is equivalent to $p(e^{\xi} + 1) = e^{\xi} p^{(u)} + p^{(l)}$, we have the unique inverse transformation $p(\xi) = (p^{(u)} e^{\xi} + p^{(l)})/(1 + e^{\xi})$.

Now, we can transform the maximisation problem to

$$\max_{\xi \in R_m} f(\xi)$$

and use an unconstrained optimisation algorithm to find $f(\hat{\xi}) = \max_{\xi \in R_m} f(\xi)$. It is guaranteed

then that function evaluations at the final estimate $\hat{\mathbf{p}}(\hat{\xi})$ and at the algorithmic search path are such that the values of the original parameters (policy measures) are within their lower and upper limits.

